AOS 452 – Lab 9 Handout
Diagnosis of Vertical Motion

INTRODUCTION

In lecture, we've discussed the QG Omega Equation, which can be written as:

$$\left(\sigma \nabla^2 + f^2 \frac{\partial^2}{\partial p^2}\right)\omega = F$$

The left-hand side of this equation is, roughly speaking, a three-dimensional Laplacian of omega. As an approximation, we usually treat the operator in parentheses as a minus sign, but in principle, we could solve this elliptic partial differential equation for omega exactly. On the right-hand side is F, the "*forcing for omega*." We have discovered numerous ways of writing F, and these are listed below. Using the approximation above, we know that $F > 0$ implies forcing for ascent, while $F < 0$ implies forcing for descent.

TRADITIONAL FORCING TERMS

$$F = -\frac{R}{p}\nabla^2\left(-v_g \cdot \nabla T\right) - f_0 \frac{\partial}{\partial p}\left[-v_g \cdot \nabla\left(\zeta_g + f\right)\right]$$

In the traditional form of the QG Omega Equation, the forcing for omega is given by two terms, the Laplacian of temperature advection, and differential vorticity advection. We'll look at the Laplacian of temperature advection term first.

Here are the settings for the Laplacian of temperature advection term at 850 mb:

```
glevel = 850
gfunc  = quo(mul(-2.87,lap(adv(tmpk,geo))),pres)
scale  = 17
```

The core of this *gfunc* is `lap(adv(tmpk,geo))`, the Laplacian of the advection of temperature by the geostrophic wind. (`tmpc` could be used instead of `tmpk` since their gradients are identical). The rest tacks on the constants out front of the mathematical expression. The factor of 2.87 (instead of 287) is due to the fact that GEMPAK uses millibars as its pressure units rather than pascals. Since we want our result to be in SI units (for comparison with other diagnostics), we must convert the pressure to Pascals, i.e., multiply the pressure by 100. Since pressure is in the denominator, this is equivalent to dividing the whole term by 100, and to simplify *gfunc*, I multiply by −R/100.

Traditionally, this term has been computed at 850 mb, although any level between 900 mb and 600 mb can typically be illuminating.

Let's now examine differential vorticity advection. To prepare for writing *gfunc*, we can approximate this term as:

$$-f_0 \frac{\partial}{\partial p}\left[-v_g \cdot \nabla\left(\zeta_g + f\right)\right] \approx -f_0 \frac{\left[-v_g \cdot \nabla\left(\zeta_g + f\right)\right]_2 - \left[-v_g \cdot \nabla\left(\zeta_g + f\right)\right]_1}{p_2 - p_1} = \frac{f_0}{p_2 - p_1}\left\{\left[-v_g \cdot \nabla\left(\zeta_g + f\right)\right]_1 - \left[-v_g \cdot \nabla\left(\zeta_g + f\right)\right]_2\right\}$$

where $p_2 > p_1$.

The settings for this term in the 900–500-mb layer are then:

```
glevel = 500
gfunc  =
   quo(mul(corl,sub(adv(avor(geo),geo),adv(avor(geo@900),geo@900))),40000)
scale  = 17
```

The *glevel* setting in this case serves to simplify *gfunc*. I could have ignored *glevel* completely if I had written `geo@500` instead of just geo. The core of this term is `adv(avor(geo),geo)`, the advection of absolute geostrophic vorticity by the geostrophic wind. Note that the constant 40000 should be set to $p_2 - p_1$, which may vary if you evaluate this term in a different layer.

Traditionally, the approximation is made that the 900-mb absolute vorticity advection is negligible, so that *gfunc* can be simplified to `quo(mul(corl,adv(avor(geo),geo)),40000)`.

TRENBERTH FORMULATION

Easily stated in words as "vorticity advection by the thermal wind," this formulation can be computed either at a level, or in a layer (just like Q-vectors). First, here's the expression valid through a layer:

$$F = 2f_0\left(\frac{\partial v_g}{\partial p} \cdot \nabla \zeta_g\right)$$

To prepare for writing *gfunc*, we can approximate this term as:

$$F = 2f_0\left(\frac{\partial v_g}{\partial p} \cdot \nabla \zeta_g\right) \approx 2f_0 \frac{v_{g_2} - v_{g_1}}{p_2 - p_1} \cdot \nabla \zeta_g = \frac{2f_0}{p_2 - p_1}\left[-\left(v_{g_1} - v_{g_2}\right) \cdot \nabla \zeta_g\right] = \frac{2f_0}{p_2 - p_1}\left(-THRM \cdot \nabla \zeta_g\right)$$

where $p_2 > p_1$ and $THRM \equiv v_{g_1} - v_{g_2}$.

The GEMPAK settings for Trenberth forcing in the 700–400-mb layer are then:

```
glevel = 400:700
gfunc  = quo(mul(corl,adv(vor(geo@550),thrm)),15000)
scale  = 17
```

The geostrophic relative vorticity should be computed at 550 mb because that is the midpoint of the particular layer chosen. The core of this term is `adv(vor(geo@550),thrm)`, the advection of geostrophic vorticity by the thermal wind. The constant 15000 should be set to $\frac{p_2 - p_1}{2}$, which may vary if you evaluate this term in a different layer. Trenberth forcing is typically evaluated for some layer bounded by 1000 and 400 mb.

Turning now to *Trenberth forcing at a level*, we have this expression:

$$F=-\frac{2R}{p}\left(\hat{k}\times\nabla T\right)\cdot\nabla\zeta_g$$

This is equivalent to the previous expression for *F*. We've just used the thermal wind equation to rewrite it.

Here are the GEMPAK settings for Trenberth forcing at 600 mb:

```
glevel = 600
gfunc  = quo(mul(5.74,adv(vor(geo),kcrs(grad(tmpk)))),pres)
scale  = 17
```

The core of this term is `adv(vor(geo),kcrs(grad(tmpk)))`, the "advection" of geostrophic vorticity by $\hat{k}\times\nabla T$. The rest tacks on the constants out front. 2R is written as 5.74 for the same reasons as in the temperature advection term.

Q-VECTOR CONVERGENCE

$$F=-2\nabla\cdot Q$$

GEMPAK can compute Q-vectors both at a particular level and through a particular layer. If we want to plot both the Q-vectors and their convergence *at a level* (say, 700 mb) we can use the following settings:

```
gfunc  = quo(mul(-5.74,div(qvec(thta,geo))),pres)
gvect  = qvec(thta,geo)
glevel = 700
scale  = 17/10
wind   = am2/.7//0221/.6
```

The size of the Q-vector arrows is controlled in two places. First, the second part of the *scale* parameter allows you to vary the size of the arrows over orders of magnitude. Then, the second part of the *wind* parameter can be used to fine-tune the size of your vectors. These settings will vary depending on the level and storm strength. Depending on how much area your GEMPAK window covers, you may want to set the *skip* parameter as well (but only the vector part of the *skip* parameter!) For example, `skip = /1` might work nicely.

You may be surprised by the *gfunc* setting. Shouldn't it be `mul(-2, div(qvec(thta,geo)))`? It turns out that GEMPAK defines its Q-vectors without the R/p term that we included in lecture. The *gfunc* above includes that term, which is identical to the constants in front of the "*Trenberth forcing at a level*" term.

For Q-vectors and their convergence *in a layer* (say 800–600 mb), use these settings:

```
gfunc  = mul(-2,div(qvcl(thta,geo)))
gvect  = qvcl(thta,geo)
glevel = 600:800
scale  = 14/9
wind   = am1/.6//0221/.6
```

NOTE: GEMPAK defines its "*Q-vectors in a layer*" in such a way that it is very hard to compare the magnitude of this term with the magnitudes of the other terms, so I'm not even trying in the *gfunc* above. This means that only the patterns of Q-vector convergence in a layer can be compared to the

patterns seen in the other terms; their actual values cannot be compared. After all, this term is three orders of magnitude larger than the others, based on the *scale* setting.

In the lab assignment, you'll analyze some of these vertical motion diagnostics.

WEB EXAMPLES

Examples of the various vertical motion diagnostics are linked from the AOS 452 webpage. (marrella.aos.wisc.edu/aos452) You may want to try to reproduce these plots on your own. (These diagnostics were provided by Steve Decker.)

One "trick" I employed to produce less noisy plots was to use the SM9S function in GEMPAK. For example, for the Laplacian of temperature advection, I used:

```
gfunc = sm9s(quo(mul(-2.87,lap(adv(tmpk,geo))),pres))
```

Roughly speaking, this tells GEMPAK to filter out structures smaller than six grid points across. From lecture, we know the quasigeostrophic assumption breaks down at small scales anyway, so this filtering is justified theoretically, as well as by the fact that it produces nicer plots.