AOS 100/101
Spring 2019
HMWK #1
Solutions

**1)** This problem involves consideration of the kinetic energy (KE) of the two balls and the fact that the KE is a measure of the work that each ball can do. First, we consider the KE of each ball. Both A and B are made of the same material so their densities ($\rho$) are the same and their respective masses are

$$\text{Mass}_A = \rho \, x \, (4/3)\pi(R_A)^3 \quad \text{and} \quad \text{Mass}_B = \rho \, x \, (4/3)\pi(R_B)^3$$

Therefore, their respective KEs are

$$KE_A = (1/2)\text{Mass}_A(V^2) \quad \text{and} \quad KE_B = (1/2)\text{Mass}_B(V^2).$$

Each ball does work on the other upon colliding. Ball A pushes Ball B a distance *D* from the point of impact. This amount of work is equal to ($\text{Mass}_B$) x (9.81) x *D*. This is also equal to Ball A's KE! Ball B pushes Ball A a distance 4D from the point of impact which represents an amount of work equal to ($\text{Mass}_A$) x (9.81) x *4D*. This is also equal to Ball B's KE. Thus we have a set of relationships we can use to figure out the answer. Since

$$(1/2)\text{Mass}_A(V^2) = \text{Mass}_B(D)9.81 \quad \text{and} \quad (1/2)\text{Mass}_B(V^2) = \text{Mass}_A(4D)9.81$$

then

$$\text{Mass}_A = (2 \, x \, \text{Mass}_B(D)9.81)/V^2 \quad \text{and} \quad \text{Mass}_B = (2 \, x \, \text{Mass}_A(4D)9.81)/V^2$$

which leads to a ratio of

$$\text{Mass}_B/\text{Mass}_A = [2 \, x \, \text{Mass}_A(4D)9.81)/V^2] \, / \, [2 \, x \, \text{Mass}_B(D)9.81)/V^2] = [4\text{Mass}_A/\text{Mass}_B].$$

Therefore

$$(\text{Mass}_B)^2 = 4(\text{Mass}_A)^2 \quad \text{or} \quad \text{Mass}_B = 2\text{Mass}_A.$$

From the first line, then,

$$\rho \, x \, (4/3)\pi(R_B)^3 = 2 \, x \, \rho \, x \, (4/3)\pi(R_A)^3 \quad \text{which leads to } (R_B/R_A)^3 = 2$$

so that $R_B = R_A \text{ x } \sqrt[3]{2}$. Since $R_B$ = 3 cm, then $R_A$ = 2.381 cm.

**2)**

Had life never evolved on Earth, the current atmospheric composition would be vastly different today. The photosynthesis reaction released enormous amounts of $O_2$ (oxygen) into the atmosphere - so much so that it now represents nearly 22% of every breath you take. The presence of so much $O_2$ resulted in the creation of the ozone layer ($O_3$). Once the ozone shield was in place, much of the deadly ultra-violet radiation from the sun was prevented from penetrating to the surface of the Earth. The presence of these two oxygen species in our atmosphere is a *direct* result of the presence of life on Earth.

**3)**

This problem involves thinking about the definition of pressure. The weight of the car is a constant and is evenly distributed over each of the 4 tires. Thus, each tire supports 480 lbs. If the tire is pressurized to 30 $lbs/in^2$ then the definition of pressure can be used to calculate the footprint.

$$\text{Pressure} = (\text{Force} / \text{Area})$$

We can rearrange this to find an expression of Area;

$$\text{Area} = (\text{Force} / \text{Pressure})$$

Thus, the area of each tire that makes contact with the ground (its footprint) is

$$\text{Area} = (480 \text{ lbs} / 30 \text{ lbs/in}^2) = 16 \text{ in}^2.$$

In January, the air inside the tire is colder yet the number of molecules inside is the same. A tire also tends to keep the same volume (though not necessarily the same shape) so the density is constant. This corresponds to case 1 in class. Since the air is colder, the average KE of the molecules of air is lower. This means the molecules move more slowly and exert less force on the inside walls of the tire in January. Consequently, the pressure they exert is lower. Using the expression derived in the first part of the question, if the tire pressure is lower, the footprint has to be larger and more of your tire makes contact with the ground. This reduces the mileage your car will get.